Experimental Comparison of Models of the Drying-Cooling Process of Flatbreads for Optimized Automated Production: the Case Study of Carasau Bread

Diego Deplano[*], **Mauro Franceschelli**[*], **Carla Seatzu**[*]

[*]Department of Electrical and Electronic Engineering, University of Cagliari, Italy

9[TH] International Conference on Control, Decision, and Information Technologies, Rome, Italy, 3-6 July 2023.

Outline

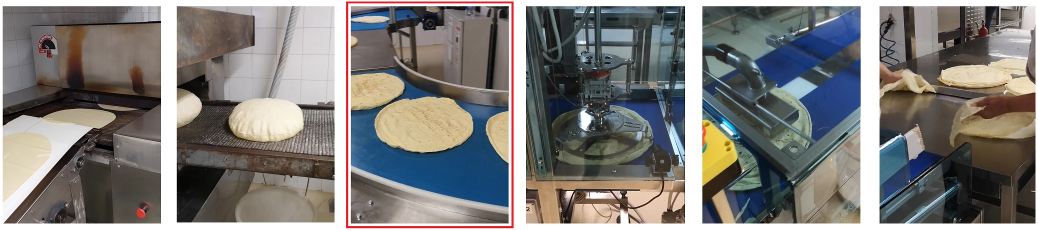
Outline

Flatbreads

Usual 3D-shaped bread

Flatbread (Carasau in the picture)

Carasau production process

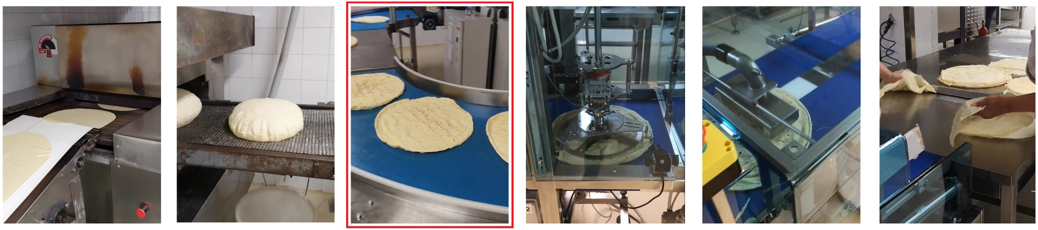


Pictures of Carasau production process showing the steps of baking in the oven, cooling/drying over the conveyor belt, and automated separation.

Problems of interest

- How to determine the correct amount of time needed by the dough to reach the desired level of temperature and moisture during the **cooling-drying process**?
- How to design and control the conveyor belt in order?

Carasau production process



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Literature review

3D-shaped bread:

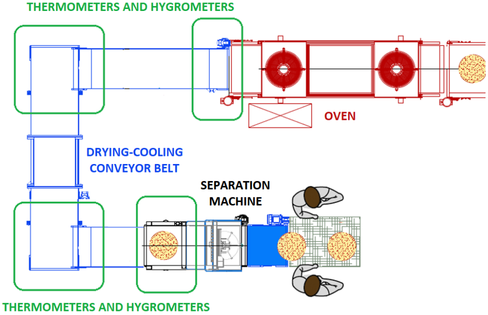
- Vodovotz, Y., L. Hallberg, and P. Chinachoti. "Effect of aging and drying on thermomechanical properties of white bread as characterized by dynamic mechanical analysis (DMA) and differential scanning calorimetry (DSC)." Cereal Chemistry 73.2 (1996): 264-270.

Flatbread:

- A. Salari, M. M. Tehrani, and S. M. Razavi, "Baking-drying kinetics of crisp bread: The influence of bran content and baking temperature," Iranian Food Science and Technology Research Journal, vol. 11, no. 3, p. 225, 2015.
- ...
- poor literature!

Outline

Problem of interest



A schematic representation of the cooling/drying process of Carasau bread, showing the steps of baking in the oven, over a conveyor belt, and automated separation.

Problem of interest

Automatization of the drying-cooling process and the subsequent separation process

Drying-cooling and separation processes

- Sheets are left to cool down and dry over a conveyor belt that brings the sheets directly into the separation machine that cut and split them apart.
- While traditionally the separation process is done manually, in our plant it has been designed a custom separation machine that automates this process.
- To ensure it functions correctly, the separation machine relies on precise levels of temperature and humidity of the sheets.

First step

Develop a model that captures the dynamics of the temperature and humidity of the sheets.

Geometric properties of the bread and assumptions during the drying-cooling process

1. Each dough has a cylindrical shape, with radius r much larger than height h, i.e., $r \gg h$.
2. Each dough is homogeneous and isotropic.
3. The material characteristics are constant and the shrinkage/expansion effect is neglected.
4. The heat transfer is much faster than the moisture transfer, thus we consider the effect of time alone on the dependent variables of moisture M and temperature T.
5. The heat and moisture transfers occur by convection between the product and the surrounding air, thus neglecting the transfer due to the conduction with the bearing surface, which is usually much slower.
6. The heat and moisture transfers occur equally at both surfaces (e.g., the dough is flipped periodically).
7. The pressure variations are neglected during the drying process.
8. Effective moisture and thermal diffusivity are constant versus moisture content and temperature during drying.

Analytical model

Let x, t be the variables modeling the space and time, respectively.

Under assumptions (1)-(8), the Luikov equations for planar geometries become

$$\frac{\partial c(x,t)}{\partial t} = D \frac{\partial c(x,t)}{\partial x^2} \tag{1}$$

where $c(x,t)$ denotes the generic local concentration of the agent (either moisture or heat) at space x and time t, and D denotes its diffusivity, assumed constant both in space and time. Initial condition

and boundary conditions:

1. Initial concentration is uniform within the product,

$$c(x,0) = c_0, \quad x \in (0,h).$$

2. Surface diffusion only happens at one surface and not on the other one, which may then be assumed to be at equilibrium,

$$c(0,t) = c_0, \quad t \geq 0,$$
$$c(h,t) = c_e, \quad t \geq 0.$$

Analytical model

Let us define the dimensionless concentration $c_r(t)$ as

$$c_r(t) = \frac{\overline{c}(t) - c_e}{c_0 - c_e}, \tag{2}$$

where $\overline{c}(t) = h^{-1} \int_0^h c(x,t)dx$ denotes the average concentration within the product at time t.

Then analytical solutions of the above equation under the considered assumptions is given by

$$c_r(t) = \sum_{i=1}^{\infty} \frac{8}{\pi^2} \frac{1}{(2i-1)^2} \exp\left[-\frac{(2i-1)^2 \pi^2 D}{4h^2} t \right]. \tag{3}$$

Approximate models

Errors in estimating the diffusion coefficient D, the equilibrium agent content c_e, and possible errors in the measurement of the dough thickness h can affect the accuracy of the moisture ratio calculation, and the combined error will depend on the number of terms considered in the series.

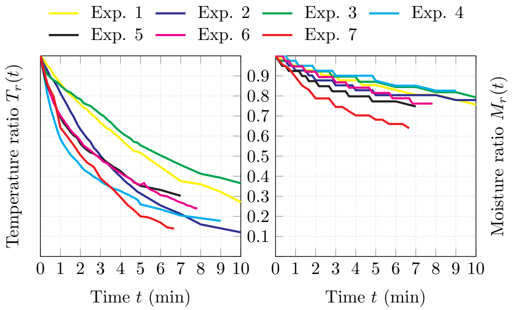
Model Name	Formula	Num. Parameters
Newton (Lewis)	$c_r(t) = \exp\left[-k_0 t\right]$	1
Logarithmic*	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)$	2
Two Term exponential	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)\exp\left[-a k_0 t\right]$	2
Diffusion Approach	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)\exp\left[-b k_0 t\right]$	3
Verma	$c_r(t) = a\exp\left[-k_0 t\right] + (1-a)\exp\left[-k_1 t\right]$	3
Noomhorm & Verma*	$c_r(t) = a\exp\left[-k_0 t\right] + b\exp\left[-k_1 t\right] + (1-a-b)$	4
Three Term exponential	$c_r(t) = a\exp\left[-k_0 t\right] + b\exp\left[-k_1 t\right] + (1-a-b)\exp\left[-k_2 t\right]$	4
Page	$c_r(t) = \exp\left[-k_0 t^n\right]$	2
Midilli and Kucuk	$c_r(t) = \exp\left[-k_0 t^n\right] + bt$	3
Modified Kaleta	$c_r(t) = a\exp\left[-k_0 t^n\right] + (1-a)\exp\left[-k_1 t^n\right]$	4
Overhults	$c_r(t) = \exp\left[-(k_0 t)^n\right]$	2
Demir*	$c_r(t) = a\exp\left[-(k_0 t)^n\right] + (1-a)$	3
Wang and Sing	$c_r(t) = 1 + at + bt^2$	2
Weibull*	$c_r(t) = (1-a) - a\exp\left[-k_0 t^n\right]$	3
Thompson	$t = a\ln(c_r(t)) + b\ln^2(c_r(t))$	2

Outline

Experimental results to select the most appropriate model

Experiments have been carried out to measure the temperature/weight decay of seven sheets under different conditions:

	Exp. 1 (yellow)	Exp. 2 (blue)	Exp. 3 (cyan)	Exp. 4 (green)	Exp. 5 (black)	Exp. 6 (magenta)	Exp. 7 (red)
Month	Feb	Feb	Feb	Feb	Jul	Jul	Jul
Ext. temperature	22°C	22°C	22°C	22°C	32°C	32°C	32°C
Init. temperature	55°C	63°C	50°C	67°C	64°C	63°C	80°C
Revolution	NO	NO	NO	NO	NO	NO	YES
Tray holes	NO	NO	YES	YES	YES	YES	YES
Tray material	Carton	Carton	Carton	Carton	Carton	Plastic	Plastic



Experimental results to select the most appropriate model

We determined the optimal parameters of each model using the function fit in Matlab that implements the Nonlinear Least Squares method.

	Temperature			Moisture		
Model name	**Min Error**	**Mean Error**	**Std. Dev.**	**Min Error**	**Mean Error**	**Std. Dev.**
Newton (Lewis)	0.0084	0.0375	0.0364	0.0056	0.0110	0.0051
Logarithmic	0.0068	0.0192	0.0176	0.0030	0.0034	0.0003
Two Term exponential	0.0069	0.0227	0.0218	0.0033	0.0068	0.0036
Diffusion Approach	**0.0038**	**0.0085**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Verma	**0.0038**	**0.0085**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Noomhorm & Verma	**0.0036**	**0.0083**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Three Term exponential	**0.0036**	**0.0080**	**0.0062**	**0.0026**	**0.0032**	**0.0004**
Page	0.0064	0.0127	0.0106	0.0028	0.0040	0.0013
Midilli and Kucuk	0.0064	0.0124	0.0102	0.0028	0.0040	0.0013
Modified Kaleta	0.0064	0.0090	0.0065	0.0027	0.0035	0.0006
Overhults	0.0064	0.0127	0.0106	0.0028	0.0040	0.0013
Demir	0.0064	0.0124	0.0111	0.0027	0.0032	0.0004
Wang and Sing	0.0110	0.0429	0.0451	0.0032	0.0047	0.0012
Weibul	0.0064	0.0124	0.0111	0.0027	0.0032	0.0004
Thompson	0.2643	0.3297	0.0542	0.2618	0.3230	0.0415

Experimental results to select the most appropriate model

The simplest of these models is the Verma model, which is a model of order 2 with 3 parameters: $k_0, k_1 \geq 0$ are the modes of the system and the coefficient $a \in (0,1)$ quantifies which mode is prevalent. The same model is suitable for both temperature and moisture decay:

$$\begin{aligned} T_r(t) &= a_T \exp\left[-k_{0,T} t\right] + (1 - a_T) \exp\left[-k_{1,T} t\right], \\ M_r(t) &= a_M \exp\left[-k_{0,M} t\right] + (1 - a_M) \exp\left[-k_{1,M} t\right]. \end{aligned} \tag{4}$$

	Param.	**Exp. 1 (yellow)**	**Exp. 2 (blue)**	**Exp. 3 (green)**	**Exp. 4 (cyan)**	**Exp. 5 (black)**	**Exp. 6 (magenta)**	**Exp. 7 (red)**
	a_T	≈ 1	≈ 1	≈ 1	0.563	0.738	0.754	0.888
$\boldsymbol{T_r(t)}$	$k_{0,T}$	$2.1 \cdot 10^{-3}$	$3.8 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$	$2.3 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$2.5 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
	$k_{1,T}$	useless	useless	useless	$2.4 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$	$4.1 \cdot 10^{-2}$
	a_M	≈ 1	0.807	≈ 1	≈ 1	0.841	0.892	0.700
$\boldsymbol{M_r(t)}$	$k_{0,M}$	$3.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	$3.0 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$3.8 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
	$k_{1,M}$	useless	$6.7 \cdot 10^{-3}$	useless	useless	$9.4 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	$6.9 \cdot 10^{-3}$

Coveyor belt design for temperature and moisture control

We can now determine the amount of wait time t^* the bread sheet must remain over the drying/cooling conveyor belt to reach the desired temperature T^* and moisture M^*:

$$\begin{cases} a_T \exp[-k_{0,T} t^*] + (1-a_T) \exp[-k_{1,T} t^*] \leq \frac{T^* - T_e}{T_0 - T_e}, \\ a_M \exp[-k_{0,M} t^*] + (1-a_M) \exp[-k_{1,M} t^*] \leq \frac{M^*}{M_0}, \end{cases} \tag{5}$$

where T_e is the temperature of the surrounding air, T_0, M_0 are the initial temperature and moisture levels, and a_T, a_M, $k_{0,T}$, $k_{1,T}$, $k_{0,M}$, $k_{1,M}$ are the parameters of the Verma models in Eq. (4) obtained offline through curve-fitting techniques.

We consider the length of the conveyor belt to be fixed, while its speed can vary within a certain range to be determined, which is the quantity of interest to be controlled:

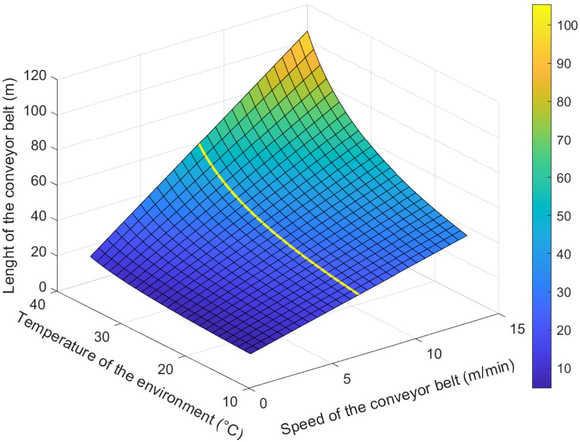
$$v_{conv} \in \left[\frac{2r}{\delta_{oven}} v_{oven},\; v_{max} \right] \tag{6}$$

where $\delta_{oven} > 2r$ is the distance between two consecutive sheets in the oven.

Length design of the conveyor belt

To determine the appropriate length, we need to consider the constraints on the speed v_{conv} of the drying/cooling conveyor belt, and the desired time t^* to achieve optimal drying-cooling:

$$\ell^* = v_{conv} t^*,$$



Temperature and moisture control

Real-time adaptive control of the conveyor belt speed can thus be used to fine tune the temperature and moisture of the bread before entering the automated separation machine when temperature and humidity of both the bread at the exit of the oven and in the immediate environment of the conveyor belt are available, e.g., thanks to thermometers and hygrometers:

$$v_{conv}(t) = \begin{cases} \frac{2r}{\delta_{oven}} v_{oven} & \text{if } \frac{t^*(t)}{\ell^*} < \frac{2r}{\delta_{oven}} v_{oven} \\ \frac{t^*(t)}{\ell^*} & \text{if } \frac{t^*(t)}{\ell^*} \in \left[\frac{2r}{\delta_{oven}} v_{oven}, v_{max}\right] \\ v_{max} & \text{if } \frac{t^*(t)}{\ell^*} \geq v_{max} \end{cases} \quad (7)$$

where $t^*(t)$ is the waiting time needed by the bread sheet arrived at time t to achieve the desired temperature and moisture levels.

The speed can be updated every 30 min or more to account for changes in the ambient temperature during the day providing closed-loop control of the drying-cooling process.

Outline

Summarizing...

- We compared different models for the water/heat transfer dynamics during the drying-cooling process of flatbreads and tested them experimentally to choose the best applicable to Carasau.
- The "Verma" model, yields the best fitting of its parameters with the available real data and has the best accuracy when used to predict the future temperature of the bread, given ambient temperature and temperature at the instant of exit from the oven.
- The proposed strategy improves the efficiency and reliability of the production process because it allows the correct design of the length and speed of the conveyor belt which transports the flatbread from the oven to the separation machine.
- Correct temperature and humidity of the flatbread at the instant of separation allow to vastly reduce product wasted because of the incorrect separation of the bread by the human operator due to excessive stickiness which leads to rupture and tears in the flat bread.

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Thank you for your attention!

Carla Seatzu